Evaluation of electrical conductivity of Cu and Al through sub microsecond underwater electrical wire explosion

D. Sheftman, D. Shafer, S. Efimov, and Ya. E. Krasik
Physics Department, Technion, Haifa 32000, Israel

(Received 28 November 2011; accepted 31 January 2012; published online 9 March 2012)

Sub-microsecond timescale underwater electrical wire explosions using Cu and Al materials have been conducted. Current and voltage waveforms and time-resolved streak images of the discharge channel, coupled to 1D magneto-hydrodynamic simulations, have been used to determine the electrical conductivity of the metals for the range of conditions between hot liquid metal and strongly coupled non-ideal plasma, in the temperature range of 10–60 KK. The results of these studies showed that the conductivity values obtained are typically lower than those corresponding to modern theoretical electrical conductivity models and provide a transition between the conductivity values obtained in microsecond time scale explosions and those obtained in nanosecond time scale wire explosions. In addition, the measured wire expansion shows good agreement with equation of state tables. © 2012 American Institute of Physics. [http://dx.doi.org/10.1063/1.3689856]

Experiments involving electrical wire explosions have been studied extensively due to the rich physical phenomena involved in this process, which results in the formation of the warm, dense matter present in the core of planets, and the generation of dense, non-ideal plasma. The interest in these plasmas is related also to confinement fusion, and solid state and plasma-chemical physics, and so on. In particular, experiments involving underwater electrical wire explosions (UEWE) were carried out extensively over recent decades. It was shown that the water environment allows one to obtain a highly uniform plasma column1,2, that enables the study of physical properties such as equations of state (EOS) and the transport parameters of the exploded material. Desiva and Katzourakis3 have carried out such experiments in the microsecond (μs) time scale, which results in a large skin depth. Thus, Ohm’s law for average electrical conductivity \( \sigma = \frac{I}{4\pi r^2 V} \), where \( l \) and \( r \) are the wire length and radius, respectively, and \( I \) and \( V \) are the discharge current and resistive voltage drop across the wire, respectively, was used to calculate the electrical conductivity for relativly thin exploding wires. However, in the sub-μs experiments, the results of which are presented in this paper, the relatively large diameter (0.25-0.8 mm) of the wires was larger than the typical skin depth. Hence, one could not use Ohm’s law to calculate \( \sigma \). Instead, one-dimensional (1D) magneto-hydrodynamic (MHD) simulations were used to obtain the electrical conductivity of the wire material using a method identical to that used in μs and nanosecond (ns) timescale experiments.1,4 Several theoretical models developed in recent years attempt to describe the physics of dense, non-ideal plasma, and in particular, the electrical conductivity. Such models include the partially ionized plasma (PIP) model,5 a modification of the Lee-More (LM) model,6 and an extension of the latter model using simulations of quantum molecular dynamics.6,7 It has been shown that these models provide good fitting to the experimental results obtained in cases of slow (typical time range \( \tau_{\text{rise}} > 1 \mu\text{s} \)) wire explosions.3 However, it was shown that during faster (time scale \( \leq 1 \mu\text{s} \)) explosions, which are characterized by higher rates of energy density deposition into the wire, the experimental conductivity does not fit any of the models for most of the densities and temperatures explored. The use of EOS tables is crucial for determining properties such as temperature and pressure in the exploding material. Indeed, it was shown that during the explosion process, a thin (a few μm) layer of plasma is formed in the water adjacent to the boundary of the wire.8,9 This plasma effectively screens the radiation from the wire boundary and, therefore, does not allow one to determine a true temperature using conventional visible spectroscopy. Hence, the EOS tables provide a method of determining the temperature indirectly. However, also EOS behavior shows a dependence on the timescale of the energy density deposition. For instance, it was shown that UEWE in ns timescale4 resulted in the evaluation of EOS values that are significantly different from those presented in SESAME tables.10 However, results of μs timescale UEWE (Refs. 3 and 4) provided good fitting to SESAME EOS tables. Thus, these results strongly suggest a dependence of the physical properties of metals at extreme conditions on external parameters, such as the rate of energy deposition into the material. The results of the experiments presented in this paper indicate that the SESAME EOS data provide a good description of the physical behavior of the wire material in the sub-μs timescale, while the conductivity values obtained still do not fit any of the conductivity models.

The experimental setup (see Fig. 1) consists of the MAGEN generator which, connected to a short circuit with an inductance of ~25 nH, provides a current pulse of ~280 kA in amplitude with a rise time of ~450 ns at a charging voltage of 60 kV. For experiments with Cu and Al exploding wires with a length of 5 cm, diameter in the range of 0.25-0.8 mm, and initial inductance of ~32 nH, the generator charging voltage was 70 kV. The total inductance of the generator and assembly (not including the wire) was ~55 nH. The wire was stretched horizontally between the
grounded and the high-voltage electrode (see Fig. 1) inside a chamber filled with de-ionized water. The top and bottom parts of the chamber have glass windows for optical observation. The discharge current and voltage were measured using B-dot and D-dot probes, respectively. A streak camera was used to obtain the expansion of the exploding wire through its self-radiation (see Fig. 2). Also, a 100 mW, 532 nm wavelength laser was used as a backlight source to obtain shadow images of the expanding wire and the strong shock wave generated in the water, and for optical calibration.

Since the skin depth of the discharge current is \( \frac{\sqrt{2}}{\pi} \frac{1}{\sigma} \approx 30 \mu m \), i.e., much smaller than the wire diameter, the wire material could not be considered radially uniform during the explosion. Therefore, 1D MHD simulations were used to reproduce the experimental discharge current and voltage waveforms and the wire radial expansion. The simulation code is described thoroughly in Ref. 1. The code uses EOS data from SESAME tables, electrical conductivity data from the conductivity model introduced by Bakulin, Luchinskii and Kuropatenkov (BKL)\(^{11}\) for Cu, and conductivity data from the QLMD model,\(^6,7\) a modification of the Lee-More-Desjarlais (LMD) model, for Al. Using a method identical to that described in Refs. 1 and 4, the input conductivity values were changed so that the simulated current and voltage waveforms would fit the experimental waveforms (see example in Fig. 3). The new conductivity values are presented in Figs. 5 and 6 in comparison with the LMD and QLMD models for temperatures of 10–60 KK, the BKL model for temperatures of up to 30 KK, and the linear response (LR) model (based on PIP calculations\(^5\)) for higher temperatures. Let us note that at conditions of thermal equilibrium, the electrical conductivity is determined by the density and temperature of the matter. Indeed, any conductivity values differing from the simulated values beyond the range of error (\( \pm 20\% \)) result in a clear mismatch between simulated and experimental waveforms. In addition, the simulated and experimental wire expansions were compared (Fig. 4). Contrary to the previous

---

**FIG. 1.** Experimental setup.

**FIG. 2.** Streak image of exploding 0.38 mm diameter Al wire (no backlighting).

**FIG. 3.** Experimental and simulated current and voltage waveforms of exploding 0.5 mm diameter Al wire. Solid lines denote experimental data and dashed lines denote simulated data.
experiments (see Refs. 1 and 4), no changes to SESAME EOS data were required in order to obtain fitting between simulated and experimental radial wire expansions.

The electrical conductivity obtained through sub-\(\mu\)s UEWE has been found to differ in value from that obtained in \(\mu\)s and ns timescale experiments. Contrary to the results of \(\mu\)s experiments, none of the models shows a consistent fitting of conductivity with the experimental results across the given range of densities and temperatures for Cu and Al materials. It was noted in Refs. 1 and 4 that differences in electrical conductivity and EOS between \(\mu\)s and ns timescale experiments could be due to external parameters such as the energy deposition rate. The conductivity results of \(\mu\)s timescale explosions are always higher than the corresponding results of ns timescale explosions (see Ref. 4) and of sub-\(\mu\)s explosions (see Fig. 7). In sub-\(\mu\)s experiments, the maximal energy density deposition rate, \(d\varepsilon/dt\), reached approximately 0.2 eV/(atom \(\cdot\) ns), while in \(\mu\)s experiments, the maximal \(d\varepsilon/dt\) reached \(~0.1\) eV/(atom \(\cdot\) ns). However, for ns timescale experiments, \(d\varepsilon/dt\) reached as much as 2.5 eV/(atom \(\cdot\) ns). It should be noted that no changes to SESAME EOS values were required for sub-\(\mu\)s experiments, and only minor changes (up to 30\%) were required for \(\mu\)s experiments. For ns experiments, the EOS values were modified significantly at high values of \(d\varepsilon/dt\). One can therefore assume that extremely high rates of energy deposition may affect the EOS behavior of metals, while moderate changes in the energy deposition rate can affect the electrical conductivity.

For Cu experiments, the sub-\(\mu\)s conductivity values showed inconsistent behavior in comparison to the other timescales (though remaining consistently smaller than \(\mu\)s conductivity values). However, for Al (see Fig. 7) at intermediate temperatures in the range of 20-30 KK, i.e., conditions of dense, warm, and non-ideal plasma, the conductivity values of the various timescales are well-ordered. Namely, \(\mu\)s values are largest, sub-\(\mu\)s values are intermediate, and ns values are smallest. At \(T=10\) KK (conditions of extremely dense, weakly ionized gas) and at \(T=40\) KK (conditions of highly ionized plasma), the ns and sub-\(\mu\)s conductivity values are similar. The intermediate temperatures of 20–30 KK are typically conditions where the maximal rate of energy deposition occurs. In addition, it can be seen in Fig. 7 that the slopes of the fitted lines of the experimental results at all timescales are similar to the QLMD model conductivity slopes at all the temperatures considered. Thus, it can be concluded that for relatively low values of \(d\varepsilon/dt\) (<0.1 eV/(atom \(\cdot\) ns)), the conductivity values for the various timescales agree with those predicted by the LMD and QLMD models and at higher energy deposition rates, and the latter models cannot provide an accurate description of the electrical conductivity. In addition, an increase in the rate of energy deposition into the material tends to shift the electrical conductivity to lower values, maintaining the dependence on material density.
FIG. 6. Electrical conductivity of Al for temperatures of (a) 10 KK, (b) 20 KK, (c) 30–40 KK, and (d) 50–60 KK. Symbols denote experimental results, the solid line denotes QLMD model, the dashed line denotes BKL model, and the dotted line denotes LR model.

FIG. 7. Comparison between conductivity values of Al for different timescales at temperatures of (a) 10 KK, (b) 20 KK, (c) 30 KK, and (d) 40 KK. Symbols denote experimental results, solid lines denote interpolated fitting of those results, and dashed lines denote QLMD model.