PHYSICS OF PLASMAS **20**, 083510 (2013)

Bohm velocity in the presence of a hot cathode

J. H. Palacio Mizrahi and Ya. E. Krasik
Physics Department, Technion, Haifa 32000, Israel

(Received 23 June 2013; accepted 2 August 2013; published online 20 August 2013)

The spatial distribution of the plasma and beam electrons in a region whose extension from a hot cathode is larger than the Debye length, but smaller than the electron mean free path, is analyzed. In addition, the influence of electrons thermionically emitted from a hot cathode and the ratio of electron-to-ion mass on the Bohm velocity and on the ion and electron densities at the plasma-sheath boundary in a gas discharge are studied. It is shown that thermionic emission has the effect of increasing the Bohm velocity, and this effect is more pronounced for lighter ions. In addition, it is shown that the Bohm velocity cannot be increased to more than 24% above its value when there is no electron emission. © *2013 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4818894]

I. INTRODUCTION

In plasma discharge with a hot cathode, the thermionic emitted electrons alter the structure of the sheath that exists between the plasma boundary and cathode wall. This occurs because a negative charge layer is formed in front of the cathode surface, while near to the plasma sheath boundary the net charge remains positive in order to confine the plasma electrons and preserve the plasma quasi-neutrality. The analysis of the structure of a sheath formed in front of a hot cathode was carried out in Refs. 1 and 2, and also addressed in Ref. 3. A space charge distribution inside the sheath formed in the front of the hot cathode is affected by contributions of thermionic electrons emitted from the cathode, the plasma ions that enter the sheath through the plasma boundary, and the plasma electrons having sufficient energy to overpass the sheath potential barrier and propagate some distance inside the sheath toward the cathode. In addition, some of these energetic plasma electrons are able to reach the cathode wall, while the others are reflected back to the plasma. The plasma electrons are usually assumed to be spatially distributed following the Boltzmann distribution function, while the electron velocities are assumed to be Maxwell distributed. In Ref. 4, the impact of a truncated Maxwell distribution of velocities for the plasma electrons on the ion dynamics was analyzed, and it was shown that in the case of low sheath potentials the ion velocity at the plasma sheath boundary can be arbitrarily small; however, this analysis concerns the case of plasma in contact with an absorbing wall and not with a hot cathode.

One of the important issues in the analysis of the ion dynamics in the region of plasma-sheath transition is the determination of the ion velocity at the plasma sheath boundary. In general, the analysis begins by assuming that the sheath has been formed; however, its existence is physically compatible only with certain ion velocities at the plasma sheath boundary. The physical condition for the sheath's existence is that a net positive charge region has to be formed just after the sheath boundary. This positively charged region creates an electric field that partially confines the plasma electrons and accelerates ions toward the wall while keeping the quasi-neutrality of the plasma. Given that the plasma quasi-neutrality is preserved until the sheath boundary is reached, the positive charge density has to decrease toward the cathode wall more slowly than the negative charge density at the sheath edge. In this analysis, neither ionization nor collisions in the sheath are considered, because its thickness is of the order of the Debye length, which is much smaller than ionization or collision mean free paths. Thus, the conservation of ion number and ion energy implies that there is a minimum value of ion velocity that leads to a decrease in the ion density more slowly than the decrease in the electron density at the sheath edge. This is the so-called Bohm criterion.[5,6] The Bohm criterion derived from physical processes in the sheath leads to a condition for the ion velocity at the plasma sheath boundary in the form of an inequality, i.e., only a lower bound for this velocity is determined; this lower bound is called Bohm velocity. The Bohm velocity is larger than the thermal velocity of the plasma ions; therefore, this velocity is obtained from an accelerating electric field in a region before the sheath boundary, which is called pre-sheath. Because the pre-sheath scale is much larger than the sheath scale, the gradient of plasma parameters in the pre-sheath presents a nonphysical singularity at the sheath boundary. Such a singularity is commonly interpreted as defining the location of the plasma sheath boundary, because from that location plasma quasi-neutrality is not satisfied, and a positive charge region begins to build up. Due to the abrupt change in the scale at the sheath boundary, the condition for sheath formation is formulated by calculating the changes of the plasma parameters with respect to changes in the plasma potential, thus avoiding the unphysical singularity. The specific form of the Bohm criterion depends on the details of the physical model that is analyzed. For example, when two or more species of ions are present in the plasma, the Bohm criterion is formulated taking into account that the net positive charge has contributions due to each type of ion.[3] The presence of more than one type of ion leads to an uncertainty concerning the velocity of each type of ion. Indeed, the Bohm criterion is related to the formation of a net positively charged region in front of the sheath edge without regard for the specific source. In the specific case of two types of ion, the problem of resolving the uncertainty in the velocities was considered in Ref. 7 using particle-in-cell simulations for an Ar/Xe mixture, and in Ref. 8 this problem was addressed by the analysis of dispersion relations for ion acoustic waves.

In this paper, a derivation of the Bohm velocity for the case of a thermionic cathode is presented, which is found by analyzing the ion dynamics in the pre-sheath. In addition, the impact of the heat exchange mechanisms at the cathode wall and the electron to ion mass ratio on the Bohm velocity and on the sheath potential is analyzed. It is found that the Bohm velocity is almost the same as the common value $\sqrt{k_B T_e/M}$ for heavy ions such as those of xenon, but can be appreciably increased with decreasing ion mass. The sheath potential is found to be always larger than the plasma floating potential, and under the same operation conditions of the gas discharge, the sheath potential increases with increasing ion mass. The boundary values of the plasma ion and electron density remain very close to each other, and only under conditions of maximum electron thermionic emission does the difference between electron and ion densities becomes non negligible. These results are important because the boundary conditions for simulation codes that use fluid models can largely be simplified.

II. DERIVATION OF THE BOHM VELOCITY

The ion dynamic in the plasma is governed by the continuity and momentum equations

$$n_i \frac{du_i}{dx} + u_i \frac{dn_i}{dx} = \dot{n}_i, \tag{1}$$

$$M\dot{n}_i u_i + M n_i u_i \frac{du_i}{dx} = -e n_i \frac{d\varphi}{dx} - M n_i \nu_{in} u_i. \tag{2}$$

Here, M, n_i, u_i are the ion mass, density, and drift velocity, respectively, e is the value of electron charge, $\dot{n}_i$ is the ion generation rate per unit of volume, φ is the plasma potential, and ν_{in} is the ion-neutral collision frequency. Equations (1) and (2) can be rewritten in matrix form

$$\begin{pmatrix} n_i & u_i \partial_\varphi n_i \\ M n_i u_i & e n_i \end{pmatrix} \begin{pmatrix} du_i/dx \\ d\varphi/dx \end{pmatrix} = \begin{pmatrix} \dot{n}_i \\ -M\dot{n}_i u_i - M n_i \nu_{in} u_i \end{pmatrix}. \tag{3}$$

In Eq. (3), it was assumed that the ion density depends on position only through the plasma potential $n_i \equiv n_i(\varphi)$. The sheath edge is defined as the location where the matrix differential equation has a singularity, i.e., the sheath edge is located at the point where the determinant of the matrix of coefficients is zero [Ref. 3, Sec. 2.3],[4] which is equivalent to

$$\frac{1}{u_B^2} = \frac{M}{e n_{i0}} \frac{\partial n_i}{\partial \varphi}\bigg|_{\varphi_0}. \tag{4}$$

Inside the sheath, the beam electrons do not collide with the plasma electrons because the mean free path for electron-electron collisions is larger than the Debye length. Once the beam electrons reach the plasma sheath boundary, they start to experience collisions with the plasma electrons, which lead to beam electron thermalization. In this region from the cathode wall up to the location at which the beam electron is thermalized, there will be two different spatial distributions, one for the plasma electrons and the other for the beam electrons. It will be assumed that the plasma electrons are Boltzmann distributed $n_e = e n_{e0} \exp\{e(\varphi - \varphi_0)/(k_B T_e)\}$, where n_{e0}, φ_0 are the plasma density and potential at the sheath edge respectively, and k_B is the Boltzmann constant. The beam electrons have a spatial distribution given by $n_{eb} = n_{eb0} f(\varphi)$, where n_{eb0} is the beam electron density at the sheath boundary and $f(\varphi)$ is a function that satisfies the boundary condition $f(\varphi_0) = 1$. The function f can be calculated in a region whose extension is shorter than the electron mean free path from the cathode wall. Such a region will include the plasma sheath boundary. In this region, the function f is determined by the continuity equation $J_{eb} = e n_{eb} u_{eb}$, where J_{eb}, n_{eb}, u_{eb} are the electron beam current density, beam electron density, and beam electron velocity, respectively. The beam electron velocity is determined by energy conservation assuming that the thermionic electrons are emitted from the cathode with negligible initial kinetic energy, i.e., $-e\varphi + m_e u_{eb}^2/2 = -e\varphi_c$, where φ_c is the plasma potential at the cathode surface and m_e is the electron mass. Thus, one obtains $e n_{eb} = J_{eb}/\sqrt{2e(\varphi - \varphi_c)/m_e}$. At the sheath boundary $e n_{eb0} = J_{eb}/\sqrt{2e\varphi_s/m_e}$, where $\varphi_s \equiv \varphi_0 - \varphi_c$ is the sheath potential. The two latter equations result in $n_{eb} = n_{eb0}\varphi_s^{1/2} (\varphi - \varphi_c)^{-1/2}$ and therefore $f(\varphi) = \varphi_s^{1/2}(\varphi - \varphi_c)^{-1/2}$. In the vicinity of the plasma sheath boundary in the plasma side, the condition of the plasma quasi-neutrality is given by

$$e n_i = e n_{e0} \exp\{e(\varphi - \varphi_0)/(k_B T_e)\} + J_{e,b}\sqrt{m_e/(2e(\varphi - \varphi_c))}. \tag{5}$$

The first term on the right hand side in Eq. (5) is the spatial distribution of the plasma electrons and the second term is the spatial distribution of the beam electrons in the region under consideration. The Bohm criterion given in Eq. (4) in this case becomes

$$u_B = \left(\frac{k_B T_e}{M}\right)^{1/2} \left(\frac{e n_{e0} + J_{e,b}\sqrt{m_e/(2e\varphi_s)}}{e n_{e0} - J_{e,b} k_B T_e \sqrt{m_e/(2e\varphi_s)^3}}\right)^{1/2}. \tag{6}$$

Let us note that Eq. (6) coincides with that found in Ref. 2. The Bohm velocity in this case is of the form $u_B = \alpha \cdot \sqrt{k_B T_e/M}$ with $\alpha \geq 1$. The Bohm criterion in this case establishes a nontrivial relation between the boundary condition for the plasma electron density and ion velocity. An important consequence of Eq. (6) is that the Bohm velocity u_B does not depend on the specific details of how the beam electrons are actually distributed. This velocity depends only on the energy of the beam electrons that they acquire in the sheath.

III. ION KINETIC ENERGY AT THE SHEATH EDGE

In the case of a non-emitting cathode, one has $J_{e,b} = 0$ and the Bohm criterion [see Eq. (6)] reduces to the well-known result $u_B = \sqrt{k_BT_e/M}$. A charge flow balance will be used in order to rewrite Eq. (6) in a form that allows us to estimate more clearly the values that u_B can take. In a gas discharge, the discharge current I_D can be calculated by a charge flow balance just in front of the cathode wall. Due to the absence of ionization and collisions in the sheath, the plasma ions reach the cathode at a rate $I_i^w/e = A_c n_{i0} u_B$, and the most energetic of the plasma electrons reach the cathode wall at a rate (Ref. 9, p. 87) $I_e^w/e = A_c n_{e0}\sqrt{k_BT_e/(2\pi m_e)}\exp[-e\varphi_s/(k_BT_e)]$. Here A_c is the cathode area. The thermionic-emitted electrons leave the cathode at a rate $A_cJ_{e,b}/e$. Therefore, the discharge current reads

$$I_D = A_c[en_{i0}u_B + J_{e,b} - en_{e0}\sqrt{k_BT_e/(2\pi m_e)}\exp(-e\varphi_s/(k_BT_e))]. \quad (7)$$

Equations (6) and (7), together with the dimensionless parameters $I_{e,b}=A_cJ_{e,b}/I_D$, $\eta_s=e\varphi_s/(k_BT_e)$, and, $\eta_0=Mu_B^2/(2k_BT_e)$ allows n_{i0}, n_{e0}, and η_0 to be written as

$$en_{e0} = \frac{I_D}{A_c}\sqrt{\frac{M}{k_BT_e}}\left\{\frac{1-I_{e,b}[1+\sqrt{m_e\eta_0/(M\eta_s)}]}{\sqrt{2\eta_0}-\sqrt{M/(2\pi m_e)}\exp(-\eta_s)}\right\}, \quad (8)$$

$$en_{i0} = \frac{I_D}{A_c}\sqrt{\frac{M}{k_BT_e}}\left\{\frac{1-I_{e,b}[1+\exp(-\eta_s)/\sqrt{4\pi\eta_s}]}{\sqrt{2\eta_0}-\sqrt{M/(2\pi m_e)}\exp(-\eta_s)}\right\}, \quad (9)$$

$$2\eta_0 = \frac{1-I_{e,b}[1+\exp(-\eta_s)/(2\sqrt{\pi\eta_s})]}{1-I_{e,b}\{1+[1+1/(2\eta_s)]\sqrt{m_e\eta_0/(M\eta_s)}-\exp(-\eta_s)/(4\sqrt{\pi\eta_s^3})\}}. \quad (10)$$

Equation (10) is an equivalent form of the Bohm criterion, Eq. (6), in terms of the normalized ion kinetic energy $\eta_0 \equiv Mu_B^2/(2k_BT_e)$; the values that the normalized electron beam current can acquire are in the range $0 \le I_{e,b} < 1$, and, as will be shown later, $\eta_s \gg 1$. Thus, one can see from Eq. (10) that $2\eta_0$ cannot differ substantially from the unity, i.e., u_B cannot differ substantially from $\sqrt{k_BT_e/M}$. In addition, the analysis of Eq. (6) shows that $Mu_B^2/(k_BT_e) = 2\eta_0 \ge 1$. Therefore, Eq. (10) implies

$$\eta_s \ge \frac{1}{2}\ln\left(\frac{M}{2\pi m_e}\right) - \frac{1}{2}\ln(2\eta_0) \approx \eta_{fp}. \quad (11)$$

Here $\eta_{fp} = 0.5\ln[M/(2\pi m_e)]$ is the plasma floating potential. By comparing the floating potential with the sheath potential calculated in Sec. IV, one finds that in the presence of electron thermionic emission the sheath potential is in fact larger than the floating potential.

One can deduce from Fig. 1 that in the case of identical operation conditions the coefficient α in $u_B = \alpha\cdot\sqrt{k_BT_e/M}$, increases with decreasing ion mass. This implies that in the presence of a emitting hot cathode the Bohm velocity depends on the ion mass in a more involved way than $u_B \propto M^{-1/2}$. The dependences of the normalized ion energy at the sheath edge η_0 on the normalized sheath potential in the case of two different electron-to-ion mass ratios for different values of electron thermionic current are shown in Fig. 1. One can see that, when the thermionic current is less than 80% of the discharge current, the ion kinetic energy at the sheath edge is increased by less than 2% above its classical value $\eta_0 = 1/2$ [see Fig. 1(b)], and this difference from the classical value decreases with increasing ion mass [see Fig. 1(a)]. However, when the thermionic current is larger than 80% of the discharge current, the increase in ion kinetic energy becomes non negligible, especially for light ions. Similar dependencies were obtained for the ratio of plasma ion/electron densities at the sheath boundary, as shown in Fig. 2.

IV. POWER BALANCE AT THE CATHODE WALL

Using the power balance at the cathode wall, one finds new facts about the behavior of the ion kinetic energy at the sheath boundary and of the sheath potential. The most energetic of the plasma electrons reach the cathode wall with a mean energy of $2k_BT_e$ per electron (see Ref. 9, pp. 467–469). After reaching the wall, each electron falls in a potential well determined by the work function φ_{wf} of the thermo-emitter material, thus releasing an energy $e\varphi_{wf}$, which results in

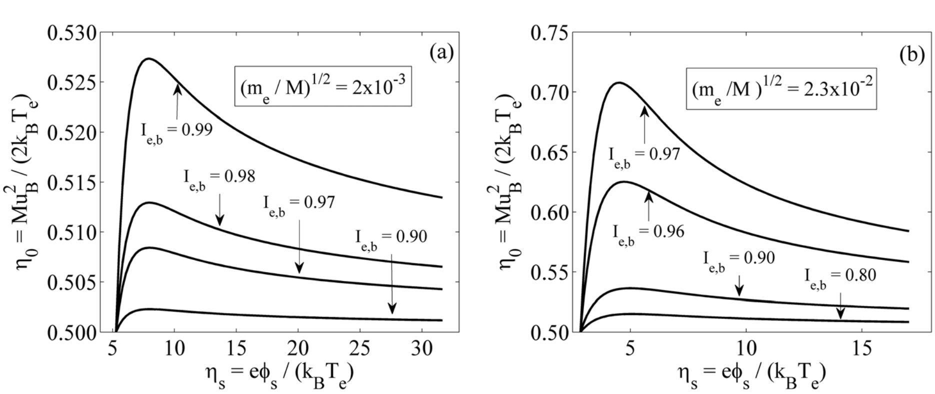

FIG. 1. Dependence of the normalized ion energy η_0 at the sheath edge on the normalized sheath potential η_s for different normalized electron beam current $I_{e,b} = A_cJ_{e,b}/I_D$ and two different electron-to-ion mass ratios: (a) xenon and (b) hydrogen.

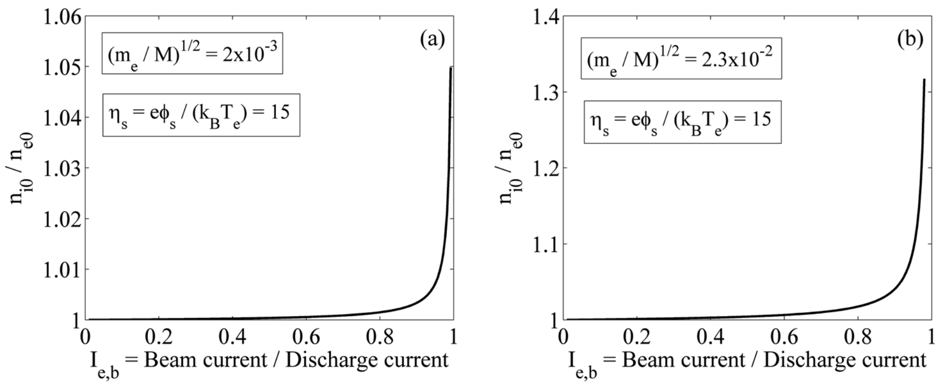

FIG. 2. Dependence of ion-to-electron density ratio at the sheath edge on normalized electron beam current for two different electron-to-ion mass ratios: (a) xenon and (b) hydrogen.

heating of the thermo-emitter. The total power P_e that the electrons deposit to the wall is $P_e = (2k_BT_e/e + \varphi_{wf})I_e^w$. The plasma ions enter the sheath with kinetic energy given by $Mu_B^2/2$, and are accelerated in the sheath potential φ_s. The total power delivered by the ions reaching the cathode wall is therefore $(\varphi_s + Mu_B^2/2e)I_i^w$. It will be assumed that each ion reaching the cathode wall is neutralized. In the neutralization process, an energy equivalent to the ionization energy $e\varphi_{iz}$ per each ion reaching the wall is released. It will be supposed that the only mechanism of electron emission from the wall is thermionic. Therefore, the total power P_i deposited by the ions on the wall is $P_i = (\varphi_{iz} + \varphi_s + Mu_B^2/2)I_i^w$. The total power $P_T^{in} = P_e + P_i$ deposited on the wall due to ion and electron impacts is, therefore,

$$P_T^{in} = (2k_BT_e/e + \varphi_{wf})I_e^w + (\varphi_{iz} + \varphi_s + Mu_B^2/2)I_i^w. \quad (12)$$

Here, it was assumed that the energy of each ion impact is effectively transferred to the cathode. After the impact, the ion reflected from the cathode wall as a neutral atom has the temperature of the wall. The latter process cools the cathode at a rate $(k_BT_w/e)I_i^w$. In addition, the electrons extracted from the cathode wall to neutralize an ion reaching the wall are originally in a potential well of depth φ_{wf}. Thus, an amount of energy equivalent to $e\varphi_{wf}$ is extracted from the cathode wall in each neutralization process resulting in a total extracted power $\varphi_{wf}I_i^w$. In addition, the thermionic emission cools the cathode at a rate $\varphi_{wf}A_cJ_{e,b}$. The cathode also loses energy due to radiation and conduction at a rate h. This rate depends on the cathode temperature and geometry. The total energy losses of the cathode are, therefore,

$$P_T^{out} = \varphi_{wf}(I_i^w + A_cJ_{e,b}) + (k_BT_w/e)I_i^w + h. \quad (13)$$

In steady state, one has $P_T^{out} = P_T^{in}$, and therefore

$$\begin{aligned}&(\varphi_s + Mu_B^2/2e + \varphi_{iz} - \varphi_{wf} - k_BT_w/e)A_cen_{i0}u_B \\ &\quad + (\varphi_{wf} + 2kT_e/e)A_cen_{e0}\sqrt{k_BT_e/(2\pi m_e)} \\ &\quad \times \exp(-e\varphi_s/k_BT_e) - A_c\varphi_{wf}J_{e,b} - h = 0. \quad (14)\end{aligned}$$

Equation (14) gives additional limits to the plasma parameters that can be realized. In normalized form and defining the dimensionless parameters $\eta_T \equiv (e\varphi_{iz} - e\varphi_{wf} - k_BT_w)/(k_BT_e)$, $H \equiv eh/(k_BT_eI_D)$, $\tilde{n}_{i0} \equiv n_{i0}(eA_c/I_D)\sqrt{kT_e/M}$, and $\tilde{n}_{e0} \equiv n_{e0}(eA_c/I_D)\sqrt{kT_e/M}$, Eq. (14) can be rewritten as

$$\begin{aligned}&(\eta_s + \eta_0 + \eta_T)\tilde{n}_{i0}\sqrt{2\eta_0} + (\eta_{wf} + 2)\tilde{n}_{e0}\sqrt{\frac{M}{2\pi m_e}} \\ &\quad \times \exp(-\eta_s) - \eta_{wf}I_{e,b} - H = 0. \quad (15)\end{aligned}$$

Equations (10) and (15) can be solved for different values of $I_{e,b}$ and H. Here, one finds that for a given value of H, Eqs. (10) and (15) have a simultaneous solution for values of $I_{e,b}$ only in a certain range. In Figs. 3 and 4, each line corresponds to a specific value of H and each line is plotted in the maximum extension of values of $I_{e,b}$ for which a simultaneous solution of Eqs. (10) and (15) exists.

One can see that the amount of heat that is removed from the cathode limits the electron beam current, the possible values of the ion energy entering the sheath, and the sheath potential. In addition, one can see that plasma discharges with light ions are characterized by lower sheath potential η_s and larger values of η_0 than those with heavier ions.

In the case of a space charge-limited electron current, the electric field at the cathode surface is zero, and it can be

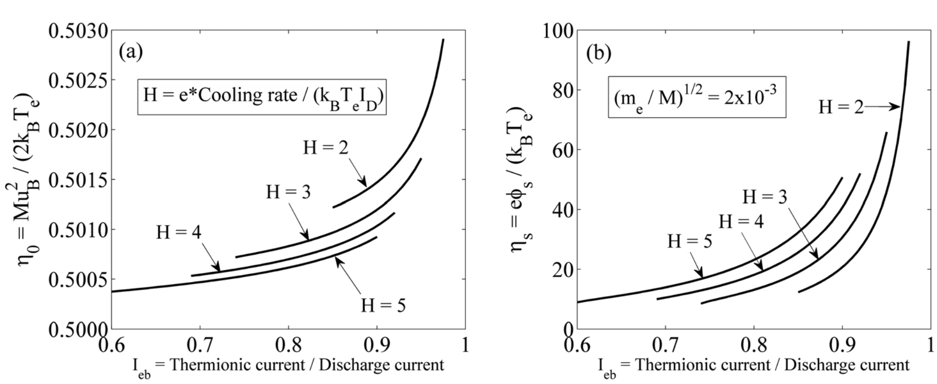

FIG. 3. (a) Normalized ion kinetic energy at the sheath boundary and (b) normalized sheath potential vs. normalized electron thermionic current for different cathode conditions for xenon plasma discharge.

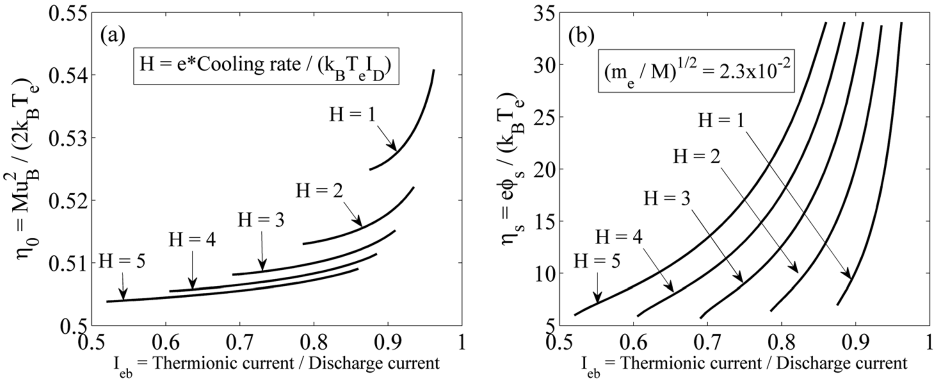

FIG. 4. (a) Normalized ion kinetic energy at the sheath boundary and (b) normalized sheath potential vs. normalized electron thermionic current under different cathode conditions for hydrogen plasma discharge.

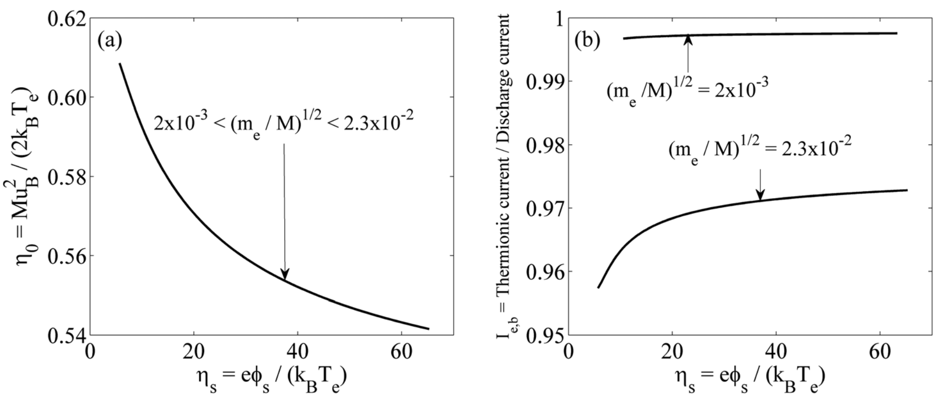

FIG. 5. (a) Normalized ion energy at the plasma sheath boundary and (b) normalized electron thermionic current vs. normalized sheath potential for different ion masses under the condition of space charge-limited electron beam current in the sheath region.

shown that (see (Ref. 2, Eq. (12) and the normalization given in the present paper)

$$\tilde{n}_{e0}\{2\eta_0[\sqrt{1+\eta_s/\eta_0}-1]+\exp(-\eta_s)-1\} + I_{e,b}\sqrt{2m_e\eta_s/M}\left\{\frac{\eta_0}{\eta_s}[\sqrt{1+\eta_s/\eta_0}-1]-1\right\}=0. \quad (16)$$

The solutions of Eqs. (10) and (16) are shown in Fig. 5. One can see that the electron-to-ion mass ratio has a negligible influence on the dependence of η_0 on η_s [see Fig. 5(a)]. It is also seen that the ion kinetic energy at the plasma sheath boundary is at most 24% larger than the common value $\eta_0 = 0.5$. Fig. 5(b) shows that in the space charge-limited case, the electron thermionic current > 96% of the discharge current.

V. CONCLUSIONS

The analysis of the spatial distribution of the plasma and beam electron densities in the region whose extension from the cathode wall is larger than the Debye length but smaller than the electron mean free path, allowed the determination of the Bohm velocity $u_B = \alpha.\sqrt{k_B T_e/M}$ which includes the effect of the thermionic emission. This expression coincides with that found by Prewett and Allen,[2] in spite of the differences in the present approach. For example, in Ref. 2, a model for the net charge $\rho(\chi)$ (where $\chi = e\varphi/k_B T_e$) in the sheath region was considered. Thus, in order to solve the Poisson equation fixed boundary conditions at the sheath edge whose location cannot be specified by the model, was assumed. In the present model the ion dynamics in the quasi-neutral plasma region was considered, and therefore, it is not necessary to solve the Poisson equation. In addition, the analysis we made specifies the location of the plasma-sheath boundary as the location at which the set of differential equations presents a singularity. It was found that the kinetic energy of the ions at the plasma-sheath boundary weakly increases in the presence of thermionic emission, and this effect decreases with increasing ion mass. The analysis of the heat transfer mechanisms at the cathode wall allowed the determination of the ion energy at the plasma sheath boundary for different operation conditions of the gas discharge. Also, it was shown that when the thermionic emission is not space charge limited, the plasma ion and electron densities have almost identical values at the plasma-sheath boundary. This result is important because boundary conditions in fluid models codes can be largely simplified. In addition, the results of this model showed that for given discharge current and cooling rate of the cathode, the sheath potential depends very sensitively on the thermionic beam current. This sensitivity increases with decreasing ion mass.

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